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Tugas 2 PDP (F)

Halaman 9

Selesaikan PDP berikut dengan metode Lagrange!

1). $2z_x - 3z_y = 4$

$\rightarrow a = 2$; $b = -3$; $c = 4$

Sehingga diperoleh persamaan karakteristik:

$$\frac{dx}{2} = \frac{dy}{-3} = \frac{dz}{4}$$

$\cdot > \dfrac{dx}{2} = \dfrac{dy}{-3}$

$-3dx = 2dy$

$\int -3dx = \int 2dy$

$-3x = 2y$

$3x + 2y = \alpha$

$\cdot > \dfrac{d}{-3} = \dfrac{dz}{4}$

$4dy = -3dz$

$\int 4dy = \int -3dz$

$4y = -3z$

$z = -\frac{4}{3}y$

$4y + 3z = \beta$

Jadi, diperoleh:

PU : $z = f(\alpha, \beta)$

$= f(3x+2y, 4y+3z)$

$z = -\frac{4}{3}y\, f(3x+2y)$

3). $az_x + bz_y = cz$

$a = a$; $b = b$; $c = cz$

Sehingga diperoleh persamaan karakteristik:

$$\frac{dx}{a} = \frac{dy}{b} = \frac{dz}{cz}$$

$\cdot > \dfrac{dx}{a} = \dfrac{dy}{b}$

$bdx = ady$

$\int bdx = \int ady$

$bx = ay$

$bx - ay = \alpha$

$\cdot > \dfrac{dy}{b} = \dfrac{dz}{cz}$

$cdy = \dfrac{bdz}{z}$

$\int cdy = \int \frac{b}{z}dz$

$cy = b\ln z$

$cy - \ln z^b = \beta$

$cy = b\ln z$

$\dfrac{cy}{b} = \ln z \longrightarrow e^{cy/b} = e^{\ln z}$

$z = e^{\frac{c}{b}y}$

Jadi, diperoleh:

PU : $z = f(\alpha, \beta)$

$= f(bx - ay, cy - \ln z^b)$

$z = e^{yc/b} f(bx - ay)$

5). $xz_x - yz_y = z$

$\rightarrow a = x$; $b = -y$; $c = z$

Sehingga, diperoleh persamaan karakteristik:

$$\frac{dx}{x} = \frac{dy}{-y} = \frac{dz}{z}$$

$\cdot > \dfrac{dx}{x} = \dfrac{dy}{-y}$

$\int \frac{dx}{x} = \int -\frac{dy}{y}$

$\ln x = -\ln y$

$\ln x + \ln y = \alpha$

$\ln xy = \alpha$

$\cdot > \dfrac{dy}{-y} = \dfrac{dz}{z}$

$\int -\frac{dy}{y} = \int \frac{dz}{z}$

$-\ln y = \ln z$

$\ln y + \ln z = \beta$

$\ln zy = \beta$

$z = \frac{1}{y}$

PU : $z = f(\alpha, \beta)$

$= f(\ln xy, \ln zy)$

$z = \frac{1}{y} f(\ln xy, \ln zy)$

7). $(x+y)(z_x + z_y) = z - 1$

$z_x(x+y) + z_y(x+y) = z - 1$

$\rightarrow a = (x+y)$, $b = (x+y)$, $c = (z-1)$

Sehingga, diperoleh persamaan karakteristik:

$$\frac{dx}{(x+y)} = \frac{dy}{x+y} = \frac{dz}{z-1}$$

$\rightarrow a = x+y \; ; \; b = x+y \; ; \; c = z-1$

$$\boxed{\frac{dx}{(x+y)} = \frac{dy}{(x+y)}} = \frac{dz}{z-1}$$

$\Rightarrow (x+y)dx = (x+y)dy$

$\int (x+y)dx = \int (x+y)\,dy$

$\frac{1}{2}x^2 + xy = xy + \frac{1}{2}y^2$

$\frac{1}{2}(x^2 - y^2) = \alpha$

$\Rightarrow \dfrac{dy}{x+y} = \dfrac{dz}{z-1}$

$\ln(x+y) = \ln(z-1)$

$z-1 = x+y \longrightarrow z-x-y-1 = \beta$

$z = x+y+1$

$u = f(\alpha, \beta)$

$= f\left(\frac{1}{2}(x^2-y^2),\; z-x-y-1\right)$

$= (x+y+1)\; f\left(\frac{1}{2}(x^2-y^2)\right)$

9). $xy\, z_x - x^2 z_y + yz = 0$

$a = xy \; ; \; b = -x^2 \; ; \; c = -yz$

$$\frac{dx}{xy} = \frac{dy}{-x^2} = \frac{dz}{-yz}$$

$\Rightarrow \dfrac{-x^2}{x}dx = y\,dy$

$\int -x\,dx = \int y\,dy$

$-\frac{1}{2}x^2 = \frac{1}{2}y^2$

$\frac{1}{2}(x^2+y^2) = \alpha$

$\Rightarrow \dfrac{d\cancel{y}x}{xy} = \dfrac{dz}{-yz}$

$-z\,dx = x\,dz$

$\int -\frac{1}{x}\,dx = \int \frac{1}{z}\,dz$

$-\ln x = \ln z$

$\ln z + \ln x = \beta$

$\ln zx = \beta$

$z = \frac{1}{x}$

$\Rightarrow u = f(\alpha, \beta)$

$= f\left(\frac{1}{2}(x^2+y^2),\; \ln xz\right)$

$= \frac{1}{x} f\left(\frac{1}{2}(x^2+y^2)\right)$

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